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Polarimetric Calibration of the *Ingara* Bistatic SAR

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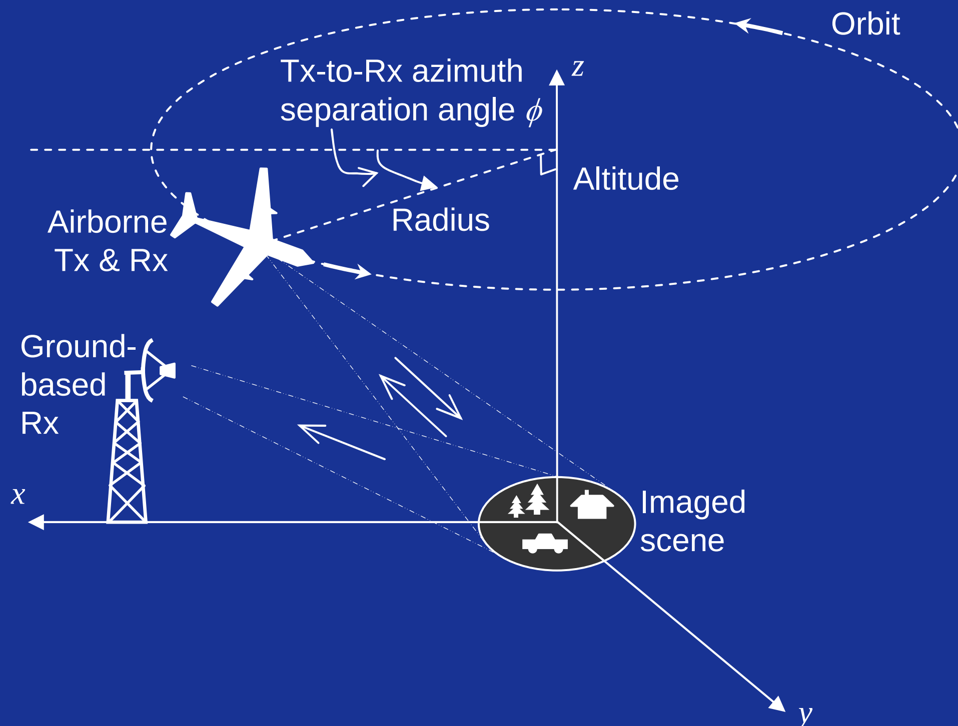
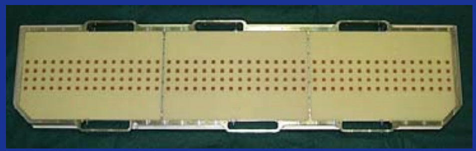
Bistatic SAR experiments

- Reciprocity $\Rightarrow s_{hv} = s_{vh}$ in monostatic but not bistatic: potentially more information in bistatic measurements
- Supplement *Ingara* X-band full-pol. airborne SAR with stationary full-pol. ground-based receiver on 15 metre high tower
- Synch. using GPS 1PPS; operate at fixed 650 Hz PRF
- Operate in circular spotlight-SAR mode: orbit radii 3 – 6 km; altitudes 1000 – 3600 m; incidence angles 53° – 82°
- Simultaneously collect 600 MHz bandwidth full-pol. monostatic and bistatic data over wide variety of angles




Beechcraft 1900C


Airborne Tx & Rx antenna


Ground-based
Rx-only antenna

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Polarimetric measurement model

Polarimetric measurement model is

$$\mathbf{O} = \mathbf{R}\,\mathbf{S}\,\mathbf{T} + \mathbf{N}$$

in which

$$\mathbf{O} = \begin{bmatrix} o_{hh} & o_{hv} \\ o_{vh} & o_{vv} \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} r_{hh} & r_{hv} \\ r_{vh} & r_{vv} \end{bmatrix} \quad \mathbf{S} = \begin{bmatrix} s_{hh} & s_{hv} \\ s_{vh} & s_{vv} \end{bmatrix} \quad \mathbf{T} = \begin{bmatrix} t_{hh} & t_{hv} \\ t_{vh} & t_{vv} \end{bmatrix} \quad \mathbf{N} = \begin{bmatrix} n_{hh} & n_{hv} \\ n_{vh} & n_{vv} \end{bmatrix}$$

Measurements — *Receive distortion* — *Target scattering* — *Transmit distortion* — *Noise*

By rewriting **O**, **S**, **N** as $\mathbf{o} = (o_{hh}, o_{hv}, o_{vh}, o_{vv})^T$, $\mathbf{s} = (s_{hh}, s_{hv}, s_{vh}, s_{vv})^T$, $\mathbf{n} = (n_{hh}, n_{hv}, n_{vh}, n_{vv})^T$, we obtain

$$\mathbf{o} = \mathbf{P}\,\mathbf{s} + \mathbf{n}$$

where

$$\mathbf{P} = Y\,\mathbf{M}\,\mathbf{A}\,\mathbf{K}$$

$$\mathbf{M} = \begin{bmatrix} 1 & v & w & vw \\ z & 1 & wz & w \\ u & uv & 1 & v \\ uz & u & z & 1 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} k^2 & 0 & 0 & 0 \\ 0 & k & 0 & 0 \\ 0 & 0 & k & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

in which

$Y = r_{vv}\, t_{vv}$ — *Absolute scaling factor*

$k = r_{hh}/r_{vv}$ $\quad \alpha = t_{hh} r_{vv} / t_{vv} r_{hh}$ — *Channel-imbalance parameters*

$u = r_{vh}/r_{hh}$, $v = t_{vh}/t_{vv}$, $w = r_{hv}/r_{vv}$, $z = t_{hv}/t_{hh}$ — *Cross-talk ratios*

Polarimetric calibration involves estimation of k, α, u, v, w, z (leaving Y to *radiometric* calibration).

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Distributed-target methods 1

For a distributed-target, can calculate covariance matrices

$$\mathbf{C_o} = \begin{bmatrix} \langle |o_{hh}|^2 \rangle & \langle o_{hh} o_{hv}^* \rangle & \langle o_{hh} o_{vh}^* \rangle & \langle o_{hh} o_{vv}^* \rangle \\ \langle o_{hv} o_{hh}^* \rangle & \langle |o_{hv}|^2 \rangle & \langle o_{hv} o_{vh}^* \rangle & \langle o_{hv} o_{vv}^* \rangle \\ \langle o_{vh} o_{hh}^* \rangle & \langle o_{vh} o_{hv}^* \rangle & \langle |o_{vh}|^2 \rangle & \langle o_{vh} o_{vv}^* \rangle \\ \langle o_{vv} o_{hh}^* \rangle & \langle o_{vv} o_{hv}^* \rangle & \langle o_{vv} o_{vh}^* \rangle & \langle |o_{vv}|^2 \rangle \end{bmatrix} \quad \mathbf{C_s} = \begin{bmatrix} \langle |s_{hh}|^2 \rangle & \langle s_{hh} s_{hv}^* \rangle & \langle s_{hh} s_{vh}^* \rangle & \langle s_{hh} s_{vv}^* \rangle \\ \langle s_{hv} s_{hh}^* \rangle & \langle |s_{hv}|^2 \rangle & \langle s_{hv} s_{vh}^* \rangle & \langle s_{hv} s_{vv}^* \rangle \\ \langle s_{vh} s_{hh}^* \rangle & \langle s_{vh} s_{hv}^* \rangle & \langle |s_{vh}|^2 \rangle & \langle s_{vh} s_{vv}^* \rangle \\ \langle s_{vv} s_{hh}^* \rangle & \langle s_{vv} s_{hv}^* \rangle & \langle s_{vv} s_{vh}^* \rangle & \langle |s_{vv}|^2 \rangle \end{bmatrix} \quad \mathbf{C_n} = \begin{bmatrix} \langle |n_{hh}|^2 \rangle & \langle n_{hh} n_{hv}^* \rangle & \langle n_{hh} n_{vh}^* \rangle & \langle n_{hh} n_{vv}^* \rangle \\ \langle n_{hv} n_{hh}^* \rangle & \langle |n_{hv}|^2 \rangle & \langle n_{hv} n_{vh}^* \rangle & \langle n_{hv} n_{vv}^* \rangle \\ \langle n_{vh} n_{hh}^* \rangle & \langle n_{vh} n_{hv}^* \rangle & \langle |n_{vh}|^2 \rangle & \langle n_{vh} n_{vv}^* \rangle \\ \langle n_{vv} n_{hh}^* \rangle & \langle n_{vv} n_{hv}^* \rangle & \langle n_{vv} n_{vh}^* \rangle & \langle |n_{vv}|^2 \rangle \end{bmatrix}$$

whereupon

$$\mathbf{C} = \mathbf{C_o} - \mathbf{C_n} = \mathbf{P}\,\mathbf{C_s}\,\mathbf{P}^H$$

Covariance matrix of distributed-target is commonly postulated to have the simple form:

$$\mathbf{C_s} = \begin{bmatrix} \sigma_{11} & 0 & 0 & \sigma_{41}^* \\ 0 & \beta & \beta & 0 \\ 0 & \beta & \beta & 0 \\ \sigma_{41} & 0 & 0 & \sigma_{44} \end{bmatrix} \quad \textbf{from} \quad \begin{cases} s_{hv} = s_{vh} \text{ (scattering reciprocity)} \\ \langle s_{ii} s_{ij} \rangle = \langle s_{ii} s_{ji} \rangle = 0 \text{ (azimuthal symmetry)} \end{cases}$$

This provides the basis for calculating α, u, v, w, z from an input covariance matrix **C**:

Ainsworth TL, Ferro-Famil L, Lee J-S, 2006, “Orientation angle preserving *a posteriori* polarimetric SAR calibration,” *IEEE Trans. Geosci. Remote Sens.*, **44**(4): 994-1003.

Klein JD, 1992, “Calibration of complex polarimetric SAR imagery using backscatter correlations,” *IEEE Trans. Aerosp. Electron. Syst.*, **28**(1): 183-94.

Lopez-Martinez C, Cortes A, Fabregas X, 2007, “Analysis and improvement of polarimetric calibration techniques,” *Proc. IGARSS 2007*, Barcelona, Spain, p. 5224-7.

Quegan S, 1994, “A unified algorithm for phase and cross-talk calibration of polarimetric data - theory and observations,” *IEEE Trans. Geosci. Remote Sens.*, **32**(1): 89-99.

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Distributed-target methods 2

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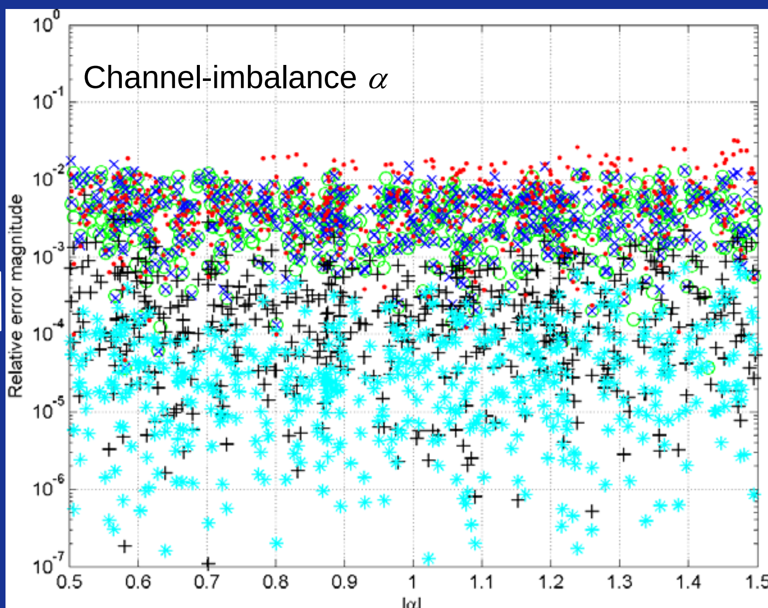
Examine a variety of methods:

- • *K*: use eigenvector of **C** (Klein 1992)
- ○ *Q*: use linear-approximations to **C** (Quegan 1994)
- × *A*: transform **C** into postulated $\mathbf{C}_s$ form while accommodating non-zero azimuthal slope (Ainsworth *et al.* 2006)
- ✳ *Ka*: *K* with improved estimation of α
- □ *Qi*: *Q* with iterations (Lopez-Martinez *et al*. 2007)
- \+ *Az*: *A* with enforced azimuthal symmetry

Compare accuracy using numerical simulations:
- Set $\sigma_{11} = \sigma_{44} = 0$ dB, $|\sigma_{41}| = -1$ dB, $\beta = -12$ dB, $Y = k = 1$
- Randomly assign $0 < |u|, |v|, |w|, |z| < 0.1$, $0.5 < |\alpha| < 1.5$, $0 \le \angle\sigma_{41}, \angle\alpha, \angle u, \angle v, \angle w, \angle z < 2\pi$

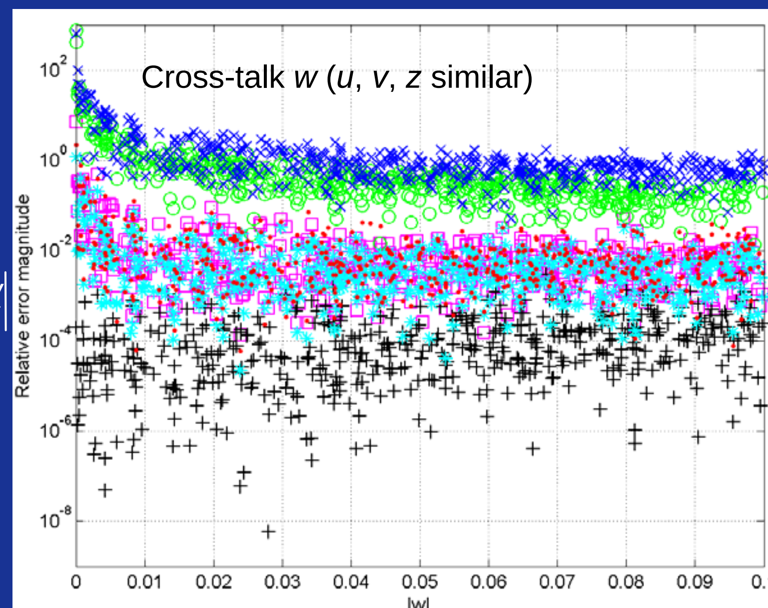
Results with full noise-compensation, i.e. $\mathbf{C} = \mathbf{C}_o - \mathbf{C}_n$

$\left|\frac{\hat{\alpha} - \alpha}{\alpha}\right|$



Ka has lowest α-estimation error

$\left|\frac{\hat{w} - w}{w}\right|$



Az has lowest cross-talk estimation error

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Calibration-target methods

Model for calibration target measurements when cross-talk is negligible (or already corrected) is

$$o_{hh} = Y\,k^2\,\alpha\,s_{hh}$$
$$o_{hv} = Y\,k\,s_{hv}$$
$$o_{vh} = Y\,k\,\alpha\,s_{vh}$$
$$o_{vv} = Y\,s_{vv}$$

(noise assumed negligible)

Hence, from measurements of a depolarising target with known s_{vh}/s_{hv}, we can estimate α via

$$\alpha = \frac{o_{vh}/o_{hv}}{s_{vh}/s_{hv}}$$

With a previously-obtained estimate of α, and measurements of a target with known s_{hh}/s_{vv}, we can estimate $\pm k$ via

$$k = \pm\sqrt{\frac{o_{hh}/o_{vv}}{\alpha\,s_{hh}/s_{vv}}} \qquad (1)$$

From measurements of a depolarising target with known s_{hh}/s_{vh} or s_{hv}/s_{vv}, we can also estimate k via

$$k = \frac{o_{hh}/o_{vh}}{s_{hh}/s_{vh}} \qquad k = \frac{o_{hv}/o_{vv}}{s_{hv}/s_{vv}} \qquad (2)$$

We can also just use (2) to resolve the sign ambiguity in (1).

For bistatic system, direct-path signal can also be used in lieu of calibration target.

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Polarimetric calibration

Distributed-target methods: *Az* *Ka*

Calibration targets

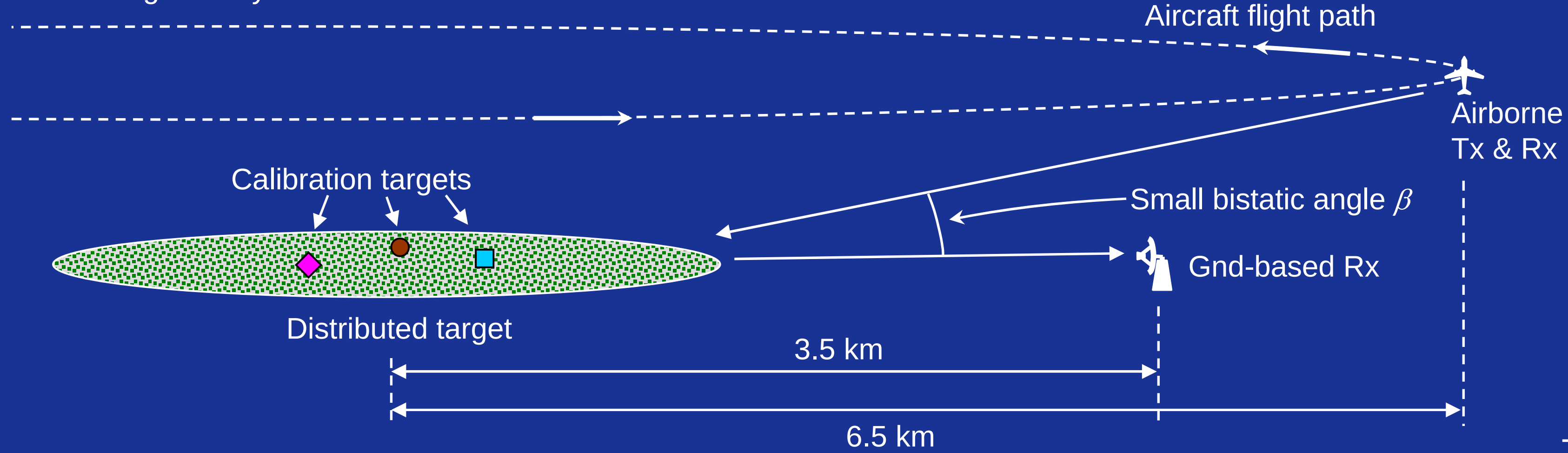
Direct-path signal

$u = r_{vh}/r_{hh}$ $v = t_{vh}/t_{vv}$ $w = r_{hv}/r_{vv}$ $z = t_{hv}/t_{hh}$

$$\alpha = \frac{t_{hh} r_{vv}}{t_{vv} r_{hh}}$$

$$k = r_{hh}/r_{vv}$$

Collection geometry for calibration data:



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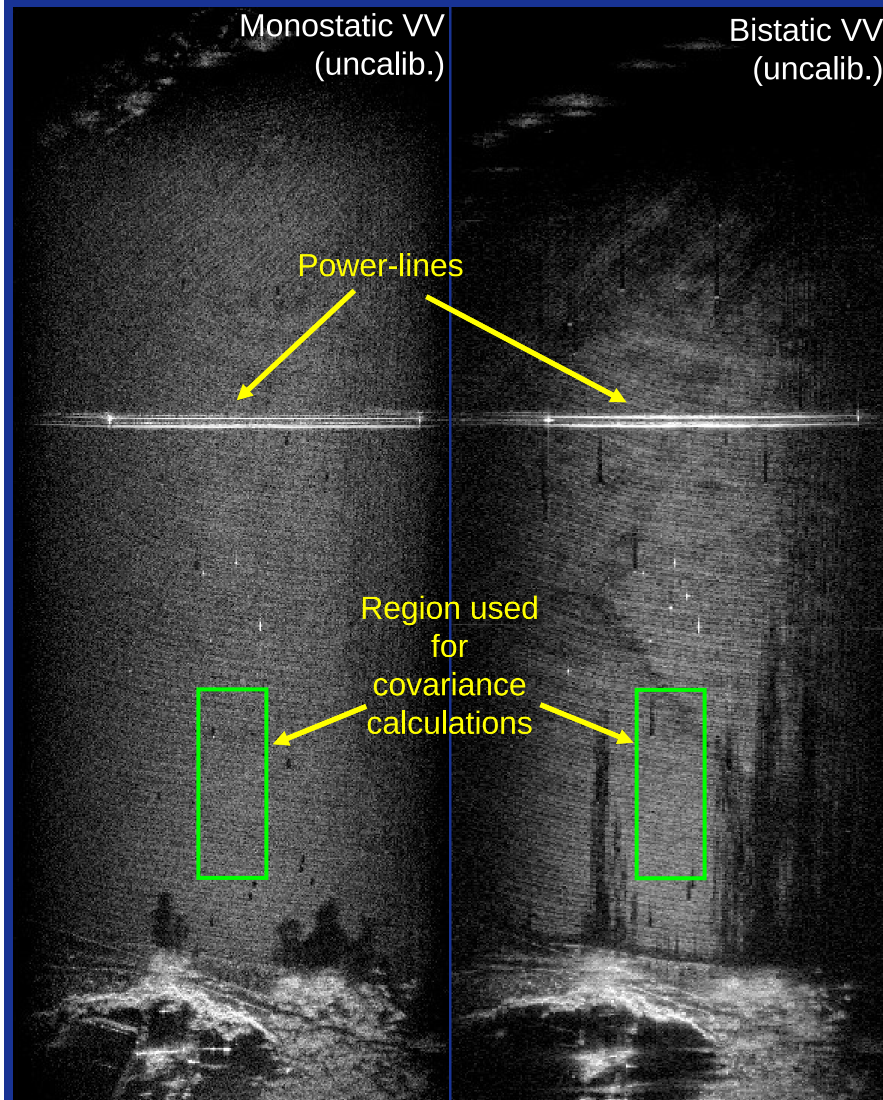
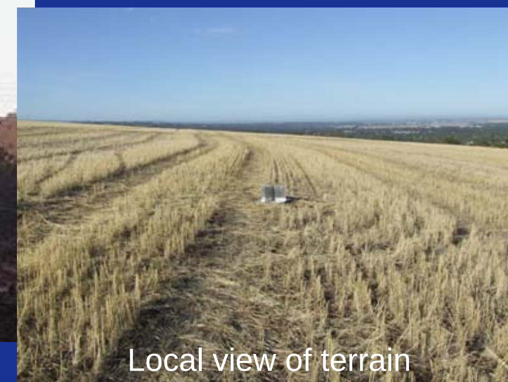
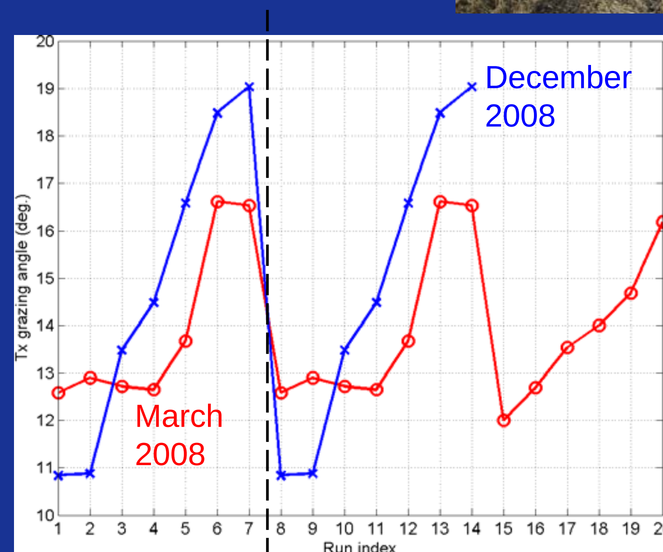
Distributed-target measurements 1



March 2008 images





Runs 1-7: Airborne Rx (monostatic)

Runs 8-20: Ground-based Rx (bistatic)

Grazing angle of ground-based Rx ≈ 2°

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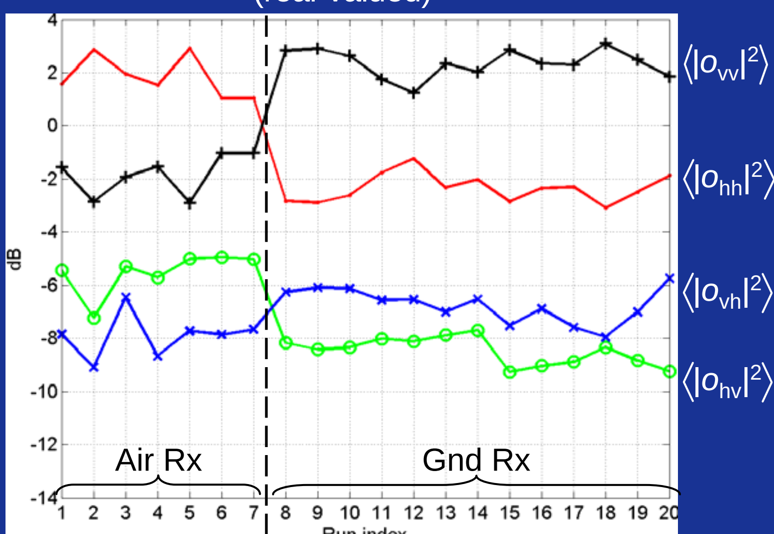
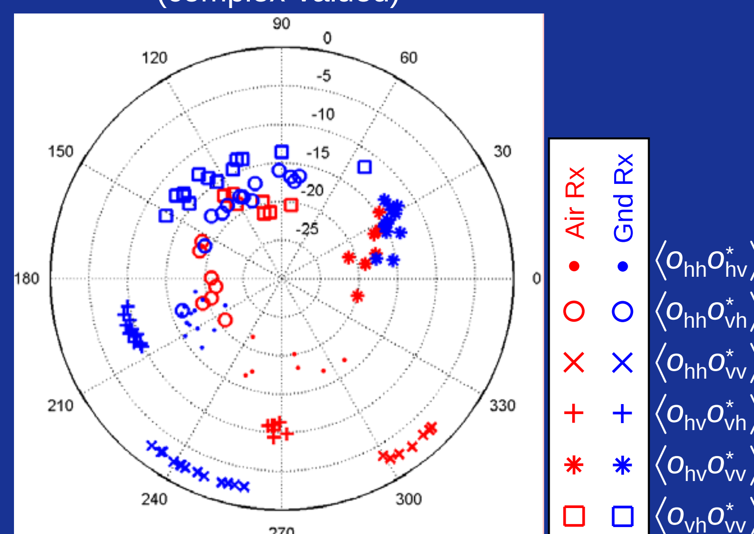
Distributed-target measurements 2

Measured covariance matrix $\mathbf{C_o} = \begin{bmatrix} \langle |o_{hh}|^2 \rangle & \langle o_{hh} o_{hv}^* \rangle & \langle o_{hh} o_{vh}^* \rangle & \langle o_{hh} o_{vv}^* \rangle \\ \langle o_{hv} o_{hh}^* \rangle & \langle |o_{hv}|^2 \rangle & \langle o_{hv} o_{vh}^* \rangle & \langle o_{hv} o_{vv}^* \rangle \\ \langle o_{vh} o_{hh}^* \rangle & \langle o_{vh} o_{hv}^* \rangle & \langle |o_{vh}|^2 \rangle & \langle o_{vh} o_{vv}^* \rangle \\ \langle o_{vv} o_{hh}^* \rangle & \langle o_{vv} o_{hv}^* \rangle & \langle o_{vv} o_{vh}^* \rangle & \langle |o_{vv}|^2 \rangle \end{bmatrix}$

On-diagonal covariances (real-valued)



Off-diagonal covariances (complex-valued)



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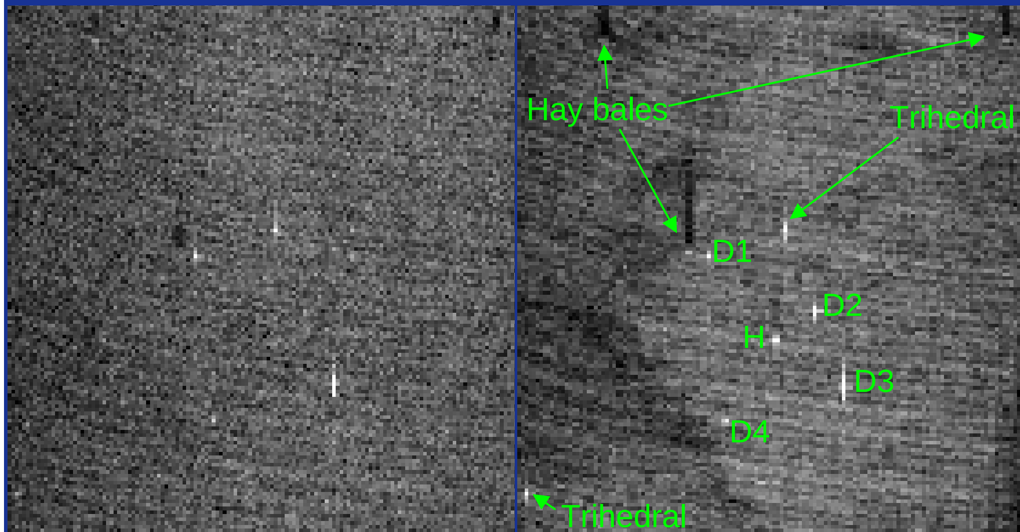
Calibration target measurements 1

Air Rx (monostatic)   Gnd Rx (bistatic)



March 2008 SAR images

Hemisphere $k = \pm\sqrt{\dfrac{\sigma_{hh}}{\alpha\,\sigma_{vv}}}$

Upright dihedral $k = \pm j\sqrt{\dfrac{\sigma_{hh}}{\alpha\,\sigma_{vv}}}$

Tilted dihedral $\begin{cases} \alpha = \sigma_{vh}/\sigma_{hv} \\ k = \pm j\sqrt{\dfrac{\sigma_{hh}}{\alpha\,\sigma_{vv}}} = \dfrac{\sigma_{hh}}{\sigma_{vh}}\tan 2\varphi = -\dfrac{\sigma_{hv}}{\sigma_{vv}}\cot 2\varphi \end{cases}$

Tilt $\varphi = -22.5^\circ$

D1: tilted 25 cm dihedral

$\mathbf{S} = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

D2: upright 25 cm dihedral

$\mathbf{S} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

H: ⌀112 cm hemisphere

$\mathbf{S} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Tilt $\varphi = +22.5^\circ$

D4: tilted 54 cm dihedral

$\mathbf{S} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

D3: upright 54 cm dihedral

$\mathbf{S} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

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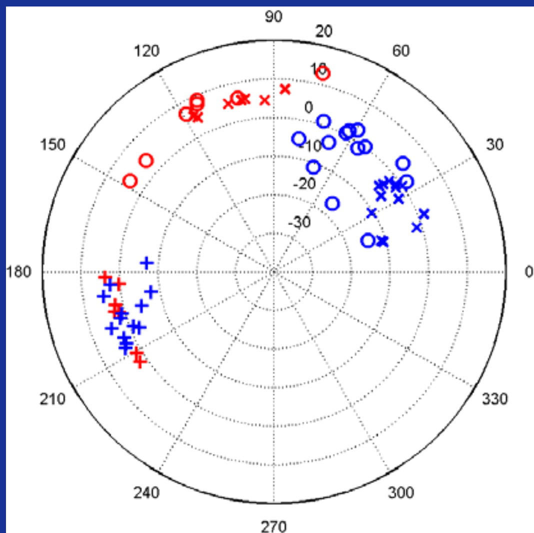
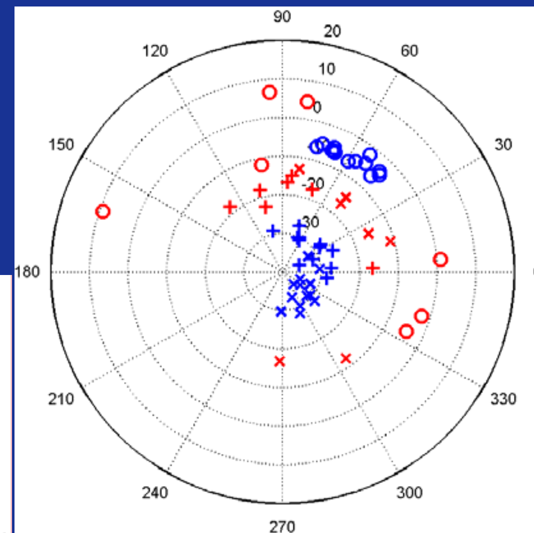
Calibration target measurements 2

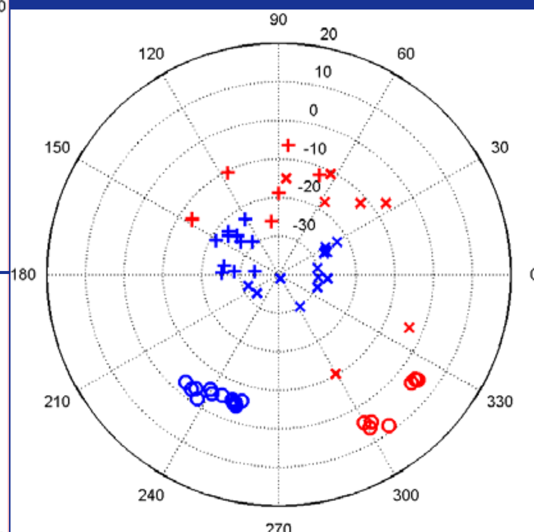


D1:
$s_{hh}/s_{vv} = -1$
$s_{hv}/s_{vv} = +1$
$s_{vh}/s_{vv} = +1$

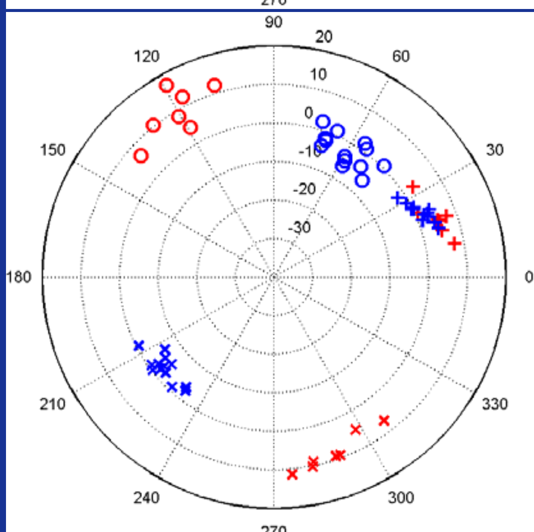
Air Rx	Gnd Rx	
O	O	o_{hh}/o_{vv}
×	×	o_{hv}/o_{vv}
+	+	o_{vh}/o_{vv}



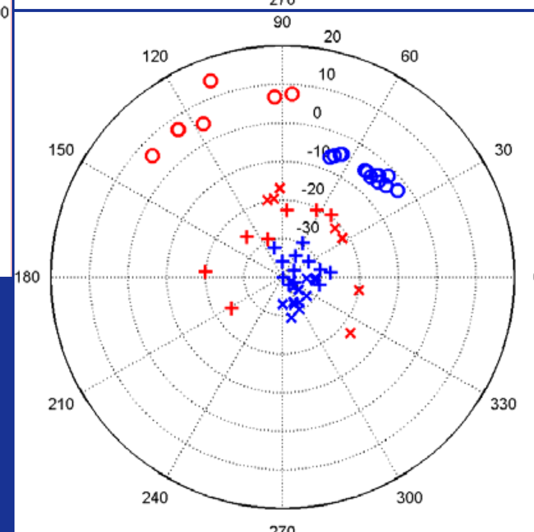
D2:
$s_{hh}/s_{vv} = -1$
$s_{hv}/s_{vv} = 0$
$s_{vh}/s_{vv} = 0$



H:
$s_{hh}/s_{vv} = +1$
$s_{hv}/s_{vv} = 0$
$s_{vh}/s_{vv} = 0$



D4:
$s_{hh}/s_{vv} = -1$
$s_{hv}/s_{vv} = -1$
$s_{vh}/s_{vv} = -1$



D3:
$s_{hh}/s_{vv} = -1$
$s_{hv}/s_{vv} = 0$
$s_{vh}/s_{vv} = 0$

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Direct-path signal measurements

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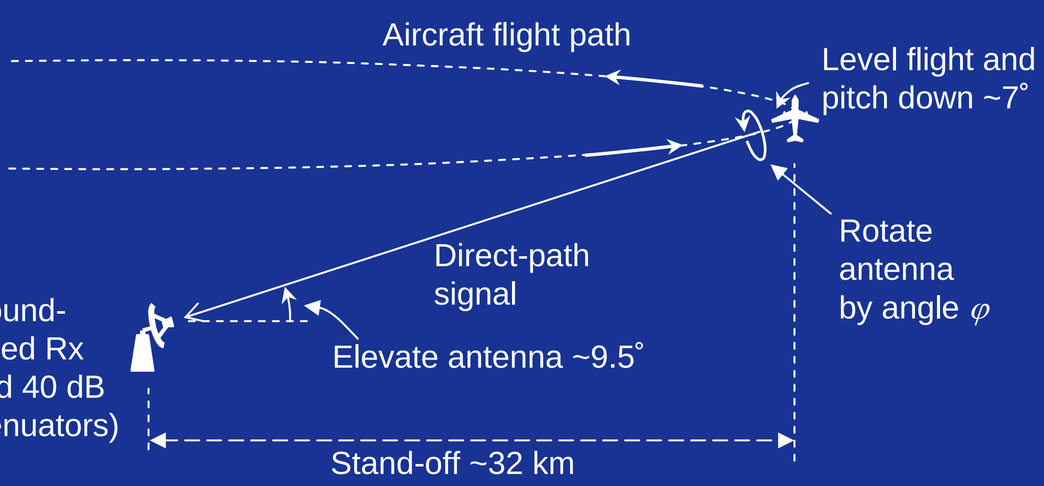
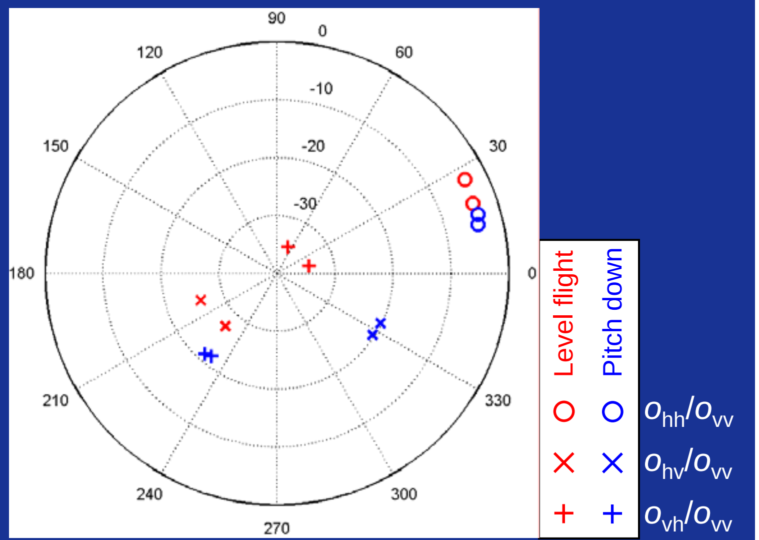
Direct-path signal measurements supplement the set of calibration target measurements.

'Scattering' matrix of direct-path signal is $\mathbf{S} = \begin{bmatrix} -\cos\varphi & -\sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$

Estimate channel-imbalance from $\begin{cases} \alpha = o_{vh}/o_{hv} \\ k = \pm j\sqrt{\dfrac{o_{hh}}{\alpha\, o_{vv}}} \end{cases}$ and/or $k = \dfrac{o_{hh}}{o_{vh}} \tan\varphi = -\dfrac{o_{hv}}{o_{vv}} \cot\varphi$





December 2008 measurements

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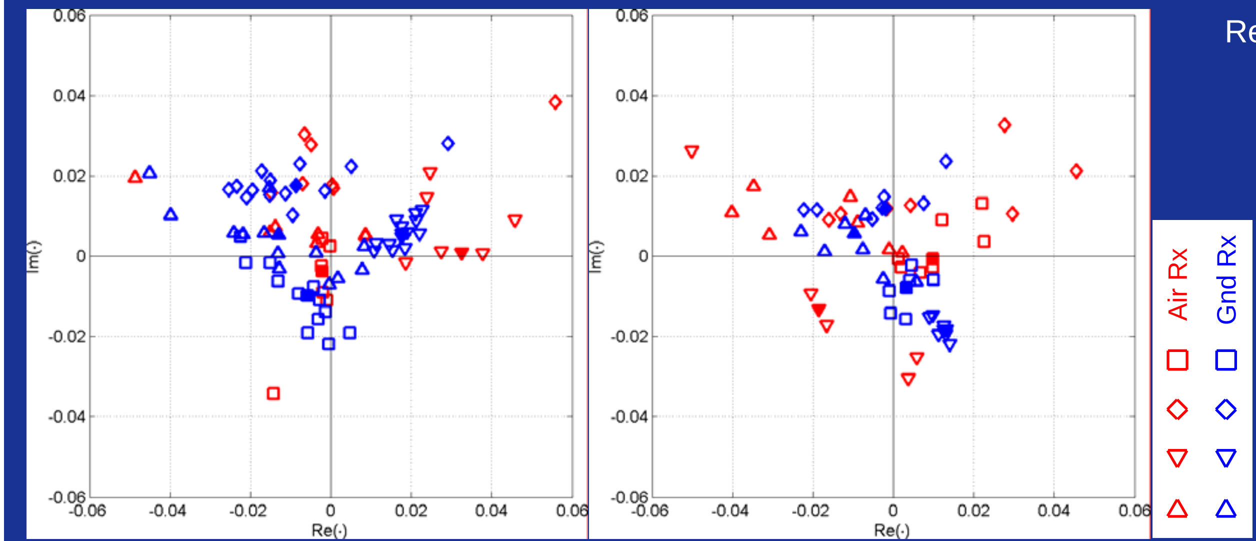
Cross-talk estimation

March 2008 data | December 2008 data



Recall: $u = r_{vh}/r_{hh}$, $v = t_{vh}/t_{vv}$, $w = r_{hv}/r_{vv}$, $z = t_{hv}/t_{hh}$

Rx only: u, w

Tx only: v, z

Expect same *v* & *z* in air and ground data but different *u* & *w*.

For each run, initial cross-talk ratio *u*, *v*, *w*, *z* estimates obtained using method *Az*.

Take 'median' of distributions to obtain final calibration parameters:

- ■ $u^{(air)}$: 'median' of *u* estimates from air data only
- ■ $u^{(gnd)}$: 'median' of *u* estimates from ground data only
- ◆ $v^{(air)}$, $v^{(gnd)}$: 'median' of pooled *v* estimates from air and ground data
- ▼ $w^{(air)}$: 'median' of *w* estimates from air data only
- ▼ $w^{(gnd)}$: 'median' of *w* estimates from ground data only
- ▲ $z^{(air)}$, $z^{(gnd)}$: 'median' of pooled *z* estimates from air and ground data

where 'median' of set of complex z_n is taken as: median({ Re(z_n) | *n* }) + j median({ Im(z_n) | *n* })

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α estimation

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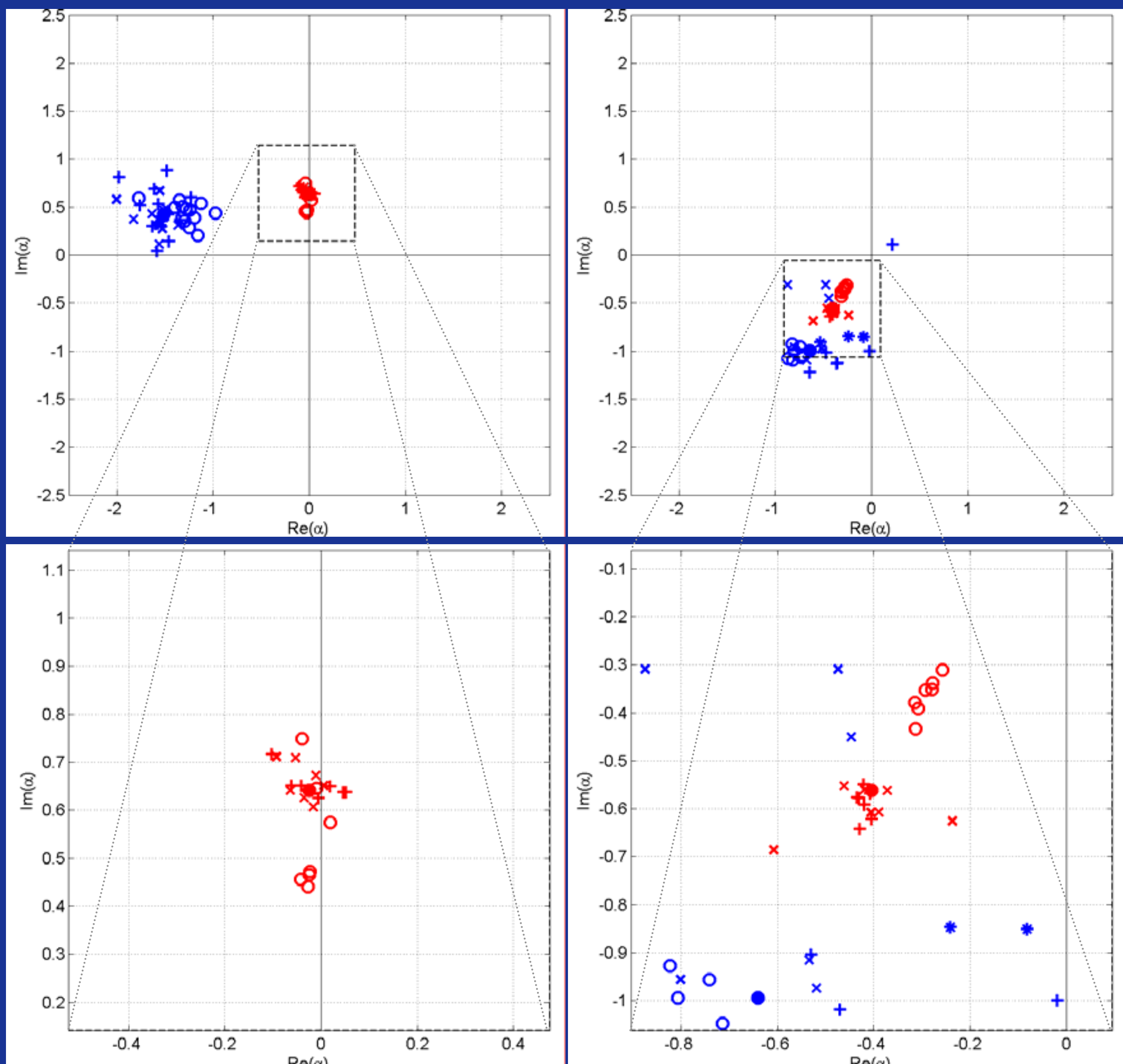
December 2008 data



Initial α estimates from individual runs

Air Rx / Ground Rx

- Distrib.-target
- D1
- D4
- Direct-path

Final calibration solution:

- $\alpha^{(air)}$: 'median' of air estimates
- $\alpha^{(gnd)}$: 'median' of ground estimates

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k estimation

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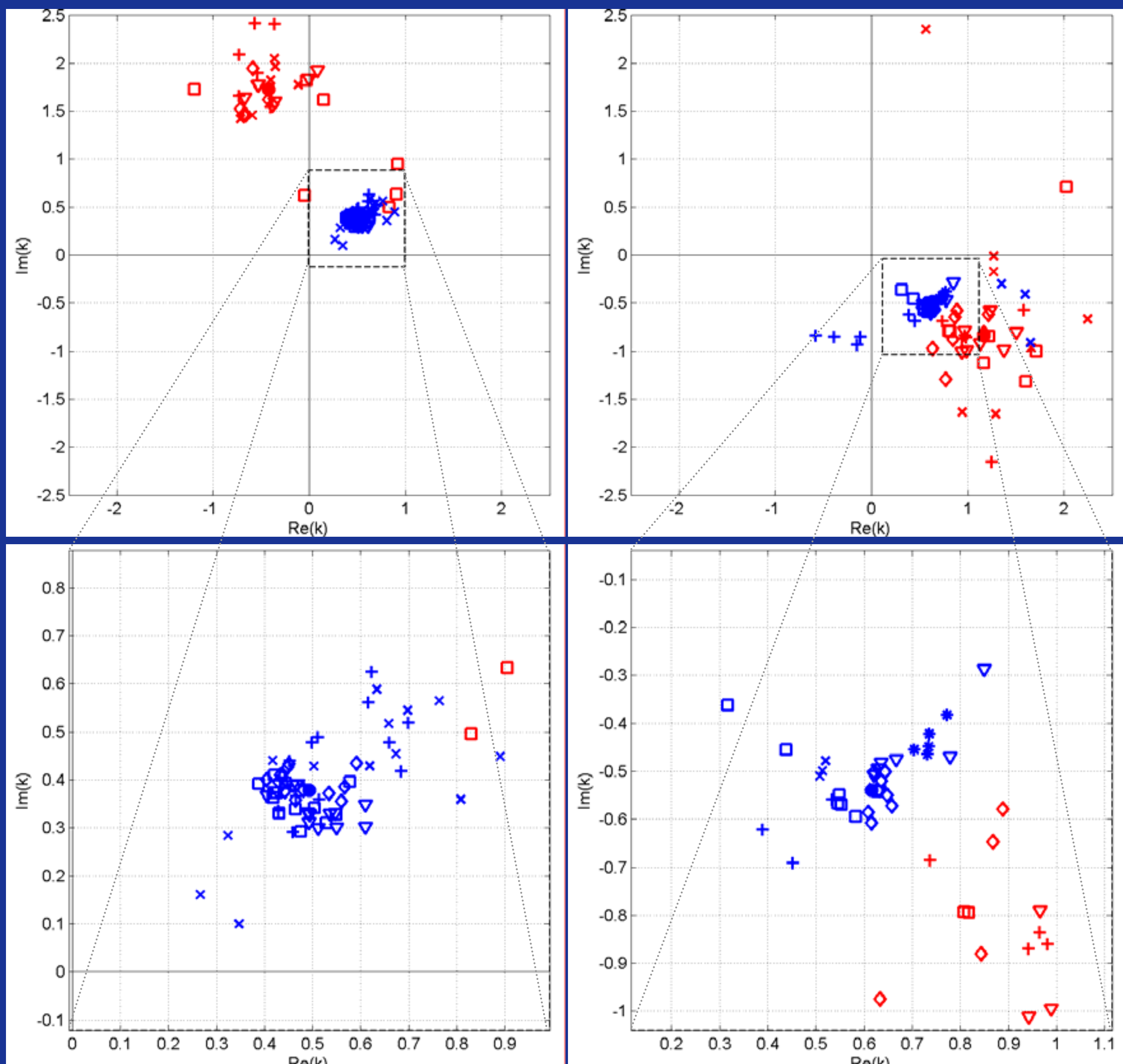
March 2008 data

December 2008 data



Initial *k* estimates from individual runs

Air Rx	Gnd Rx	
×	×	D1
□	□	D2
▽	▽	D3
+	+	D4
◇	◇	H
	✱	Direct-path

Final calibration solution:

- ● $k^{(air)}$: 'median' of air estimates
- ● $k^{(gnd)}$: 'median' of ground estimates

Post-calibration target measurements

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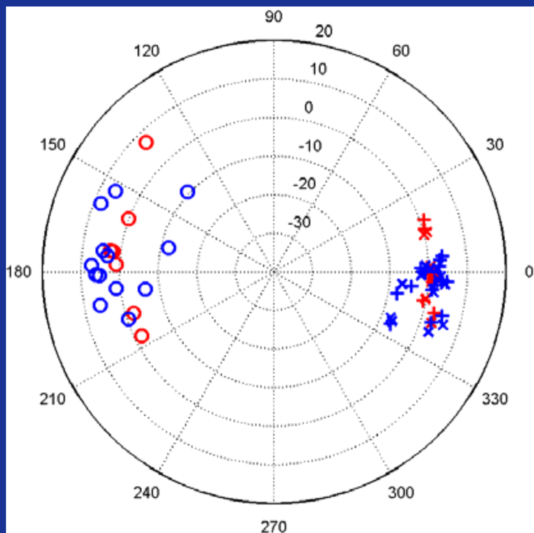
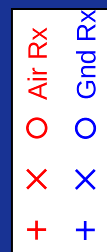
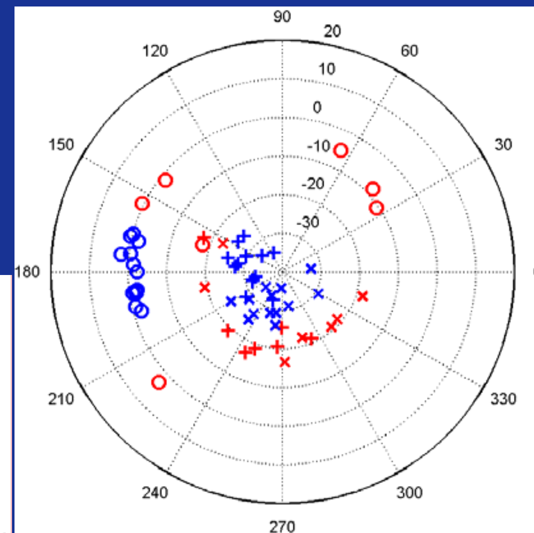


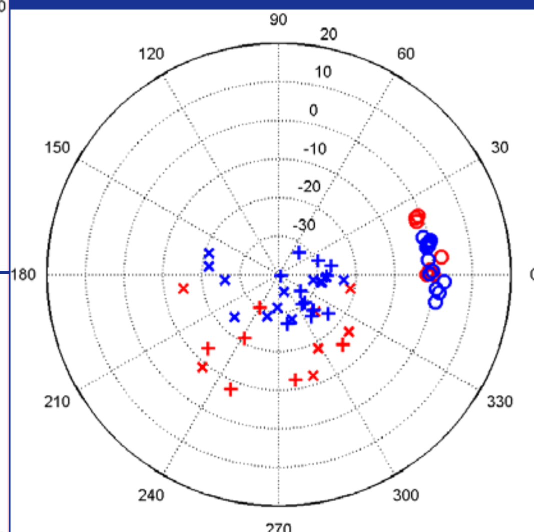
D1:
s_{hh}/s_{vv} = -1
s_{hv}/s_{vv} = +1
s_{vh}/s_{vv} = +1



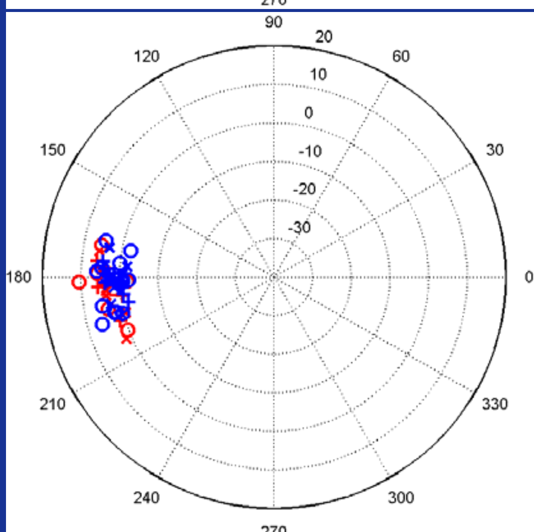
s_{hh}/s_{vv}
s_{hv}/s_{vv}
s_{vh}/s_{vv}



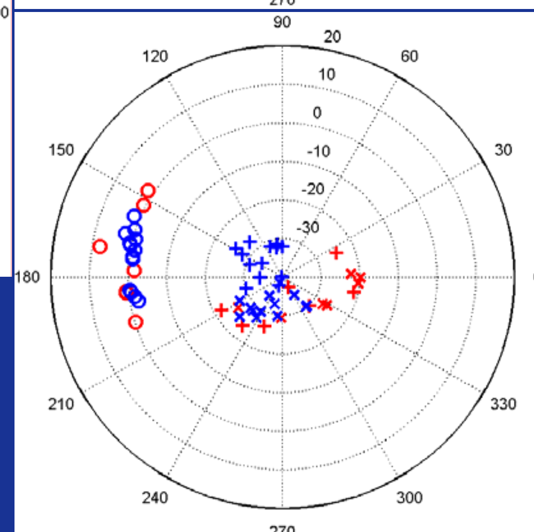
D2:
s_{hh}/s_{vv} = -1
s_{hv}/s_{vv} = 0
s_{vh}/s_{vv} = 0



H:
s_{hh}/s_{vv} = +1
s_{hv}/s_{vv} = 0
s_{vh}/s_{vv} = 0



D4:
s_{hh}/s_{vv} = -1
s_{hv}/s_{vv} = -1
s_{vh}/s_{vv} = -1



D3:
s_{hh}/s_{vv} = -1
s_{hv}/s_{vv} = 0
s_{vh}/s_{vv} = 0

March 2008 measurements

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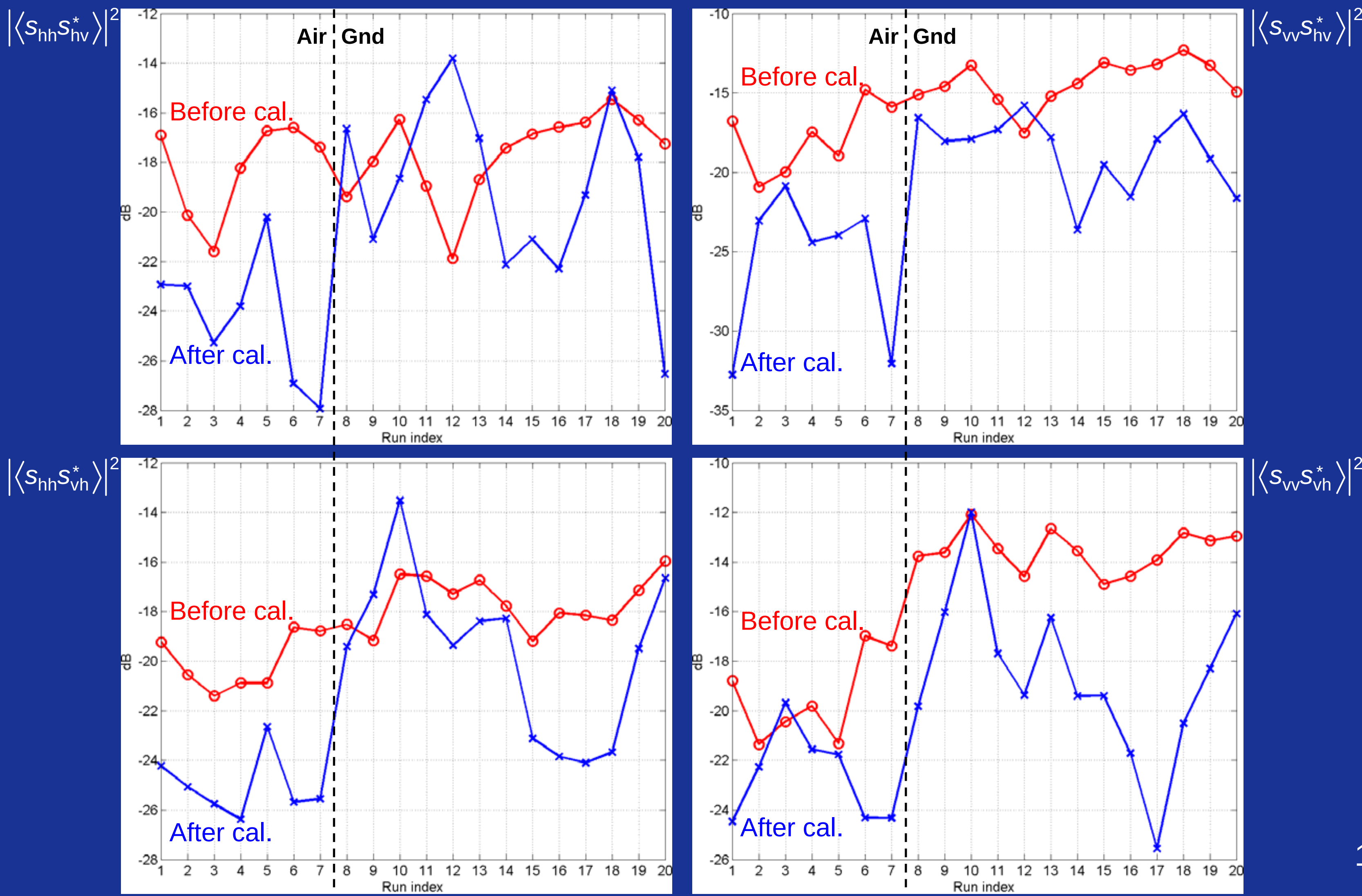
Distributed-target co-/cross-polar covariances



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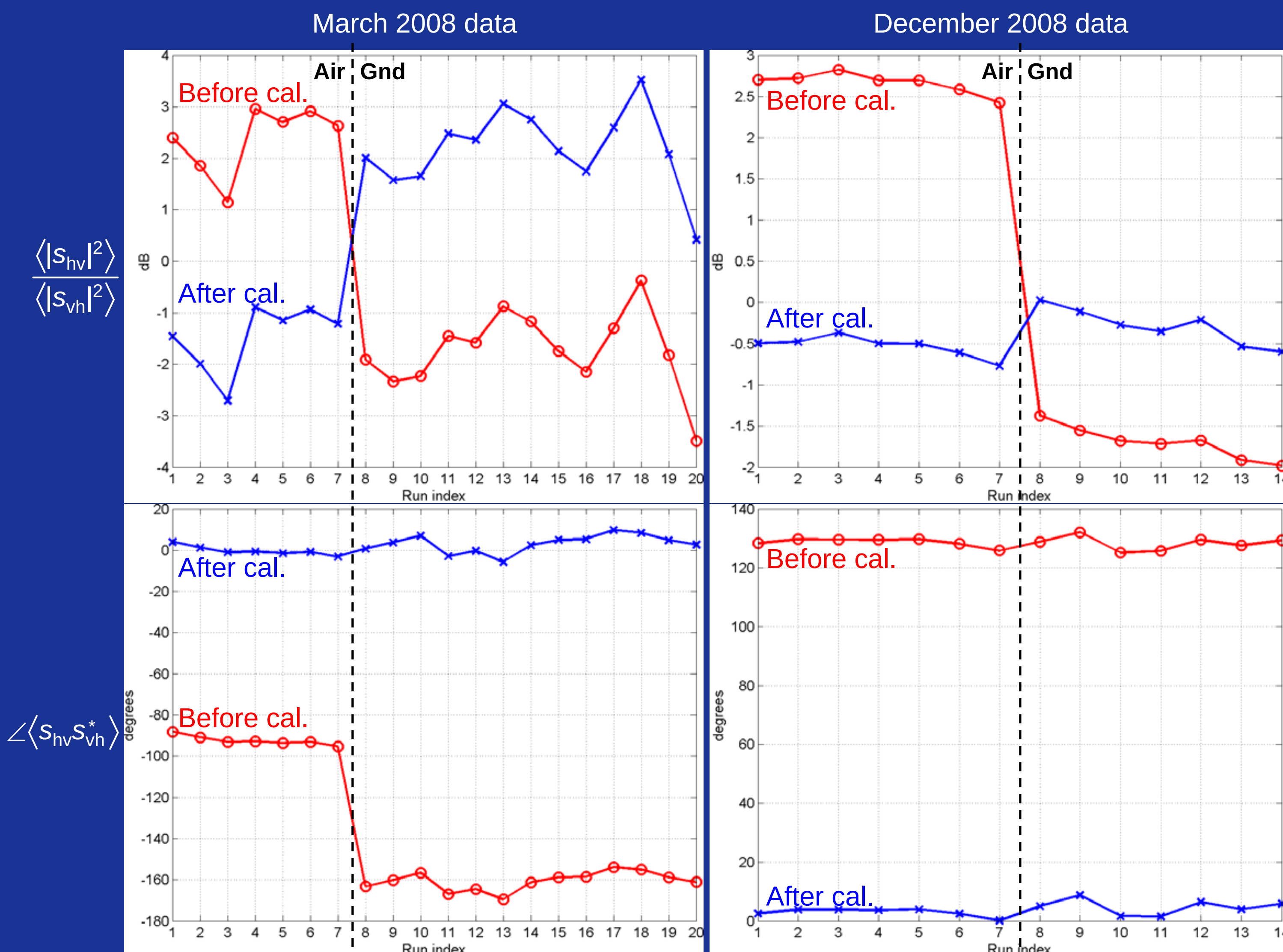
Distributed-target cross-polar measurements



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Conclusion

- A method for polarimetric calibration employing distributed-target, calibration-target and direct-path signal measurements has been applied to the *Ingara* monostatic and bistatic data.
- High variability is present in calibration target measurements possibly related to large range of look angles (e.g. bistatic from 9° to 17°): use Polarimetric Active Radar Calibrators (PARCs) in future?
- Assumptions of distributed-target azimuthal-symmetry, i.e. $\langle s_{ii}\, s_{ij}^{*} \rangle = 0$, may not be valid: validity of cross-talk calibration solution is uncertain.
- Fair agreement in α and k channel-imbalance estimates from distributed-target, calibration target and direct-path signal measurements is found.
- Application of calibration solution to measurements produces results generally more consistent with those expected of calibrated data.